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Limit Cycle Walking

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1. Introduction

This chapter introduces the paradigm `Limit Cycle Walking'. This paradigm for the design and control of two-legged walking robots can lead to unprecedented performance in terms of speed, efficiency, disturbance rejection and versatility. This is possible because this paradigm imposes fewer artificial constraints to the robot's walking motion compared to other existing paradigms. The application of artificial constraints is a commonly adopted and successful approach to bipedal robotic gait synthesis. The approach is similar to the successful development of factory robots, which depend on their constrained, structured environment. For robotic walking, the artificial constraints are useful to alleviate the difficult problem of stabilizing the complex dynamic walking motion. Using artificial stability constraints enables the creation of robotic gait, but at the same time inherently limits the performance of the gait that can be obtained. The more restrictive the constraints are, the less freedom is left for optimizing performance.

The oldest and most constrained paradigm for robot walking is that of `static stability', used in the first successful creation of bipedal robots in the early 70's. Static stability means that the vertical projection of the Center of Mass stays within the support polygon formed by the feet. It is straightforward to ensure walking stability this way, but it drastically limits the speed of the walking motions that can be obtained. Therefore, currently most humanoid robots use the more advanced `Zero Moment Point' (ZMP) paradigm (Vukobratovic et al., 1970). The stability is ensured with the ZMP-criterion which constrains the stance foot to remain in flat contact with the floor at all times. This constraint is less restrictive than static walking because the Center of Mass may travel beyond the support polygon. Nevertheless, these robots are still under-achieving in terms of efficiency, disturbance handling, and natural appearance compared to human walking (Collins et al., 2005).

The solution to increase the performance is to release the constraints even more, which will require a new way of measuring and ensuring stability. This is the core of `Limit Cycle Walking'; a new stability paradigm with fewer artificial constraints and thus more freedom for finding more efficient, natural, fast and robust walking motions. Although this is the first time we propose and define the term `Limit Cycle Walking', the method has been in use for a while now. The core of the method is to analyze the walking motion as a limit cycle, as first proposed by Hurmuzlu (Hurmuzlu and Moskowitz, 1986). Most of the research on `Passive Dynamic Walking' initiated by McGeer (McGeer, 1990a) follows this stability method. But also various actuated bipedal robots that have been built around the world fall in the category of `Limit Cycle Walkers'.

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This chapter continues as follows. In Section 2 we will give a short discussion on walking stability to create a background for the introduction of Limit Cycle Walking. The exact definition of Limit Cycle Walking follows in Section 3. In Section 4 we show how the stability of Limit Cycle Walking is assessed and clarify this with an exemplary Limit Cycle Walking model. Section 5 gives an overview of the current State of the Art Limit Cycle Walkers. In Sections 6 through 8 we will explain how Limit Cycle Walking is beneficial to the performance of bipedal walkers and substantiate this by showing the State of the Art performance. We will end with a conclusion in Section 9.

2. Bipedal walking stability

Stability in bipedal walking is not a straightforward, well defined concept (Vukobratovic et al., 2006; Goswami, 1999; Hurmuzlu et al., 2004). To create a background for the definition of Limit Cycle Walking, we discuss two extreme classifications of walking stability, the most generic one and an overly restrictive one.

The most generic definition of stability in bipedal walking is `to avoid falling'. This concept is captured with the `viability kernel' by Wieber (Wieber, 2002), the union of all viable states from which a walker is able to avoid a fall. This set of states includes all kinds of possible (periodic) motions or static equilibria and should be established with the presence of possible disturbances. Ultimately, to get optimal performance, bipedal walkers should be designed using this notion of stability, without any more restrictions. However, it turns out that it is not practical for gait synthesis due to its highly nonlinear relation with the state space of a walker. Establishing stability using this definition requires a full forward dynamic simulation or actual experiment starting out at all possible states of the walker, including all possible disturbances, checking whether this results in a fall or not. Given the complex dynamics involved in walking this would be very expensive, numerically as well as experimentally.

The limited practical value of `avoiding to fall' as a stability definition for gait synthesis, has lead a large group of robotic researchers (Sakagami et al., 2002; Ishida, 2004; Hirukawa, 2003) to create bipedal walking based on an overly restrictive classification of stability. We refer to this stability classification as `sustained local stability'. In this case, gait is synthesized as a desired trajectory through state space (usually taken from human gait analysis), which is continuously enforced by applying stabilizing trajectory control. This control aims for sustained local stability, which is obtained if for every point on the nominal trajectory it can be proven that points in its local neighbourhood in state space converge to the trajectory.

The aim for sustained local stability creates two important constraints for bipedal walking: it requires local stabilizability and high control stiffness. Local stabilizability exists when at least one foot is firmly placed on the ground. This constraint is guaranteed by satisfying the Zero Moment Point (ZMP) or Center of Pressure (CoP) criterion (Vukobratovic et al., 1970; Vukobratovic and Borovac, 2004). The constraint of high control stiffness is required to obtain local stability in spite of the presence of the inherently unstable inverted pendulum dynamics of the single stance phase.

In the strive for increasing the performance of bipedal robots, recently a growing group of researchers has decided to let go of the restrictive aim for sustained local stability and adopt a new paradigm for synthesizing bipedal gait, `Limit Cycle Walking'.

3. Definition Limit Cycle Walking

Here we formally define the new (yet popular) paradigm `Limit Cycle Walking':

Limit Cycle Walking is a nominally periodic sequence of steps that is stable as a whole but not locally stable at every instant in time.

With nominally periodic sequence of steps we mean that the intended walking motion (in the ideal case without disturbances) is a series of exact repetitions of a closed trajectory in state space (a limit cycle) putting forward each of the walker's two feet in turn. This trajectory is not locally stable at every instant in time, taking out the necessity of making all points on the trajectory attracting to their local neighbourhood in state space (as it is done in conventional trajectory control). The nominal motion is still stable as a whole because neighbouring trajectories eventually, over the course of multiple steps, approach the nominal trajectory. This type of stability is called `cyclic stability' or `orbital stability' (Strogatz, 2000).

4. Stability analysis

Cyclic stability is the core principle of Limit Cycle Walking. In this section we show how it can be analyzed. This explanation is followed by an example of a Limit Cycle Walking model which shows that it is possible to have cyclic stability without having sustained local stability.

4.1 Method

Cyclic stability of a Limit Cycle Walker is analyzed by observing its motion on a step-to-step basis. One step is considered as a function or `mapping' from the walker's state $\mathbf{v}_n$ at a definite point within the motion of a step (for instance the moment just after heel strike) to the walker's state at the same point in the next step $\mathbf{v}_{n+1}$. This mapping is generally called a Poincaré map in nonlinear dynamics and the definite point within the motion is defined by the intersection of the motion with the Poincaré section (Strogatz, 2000). With regard to walking, the mapping was termed the `stride function' **S** by McGeer (McGeer, 1990a):

$$\mathbf{v}_{n+1} = \mathbf{S}(\mathbf{v}_n) \tag{1}$$

This mapping **S** is defined by the equations of motion of the walker which are usually solved numerically and integrated over the course of one step.
A periodic motion exists if the mapping of the walker's state gives exactly the same state one step later. This specific state $\mathbf{v}^*$ is called the `fixed point' of the function **S**:

$$\mathbf{v}^* = \mathbf{S}(\mathbf{v}^*) \tag{2}$$

The cyclic stability of this periodic motion is found by linearizing the function **S** at the fixed point $\mathbf{v}^*$, assuming only small deviations $\Delta\mathbf{v}$:

$$\begin{aligned} \mathbf{S}(\mathbf{v}^* + \Delta\mathbf{v}) &\approx \mathbf{v}^* + \mathbf{A}\Delta\mathbf{v} \\ \text{with } \mathbf{A} &= \frac{\partial \mathbf{S}}{\partial \mathbf{v}} \end{aligned} \tag{3}$$

The matrix **A**, also called the monodromy matrix, is the partial derivative of the function **S** to the state **v**. Stability of the cyclic solution is assured for small state deviations if the eigenvalues λ of the matrix **A** are within the unit circle in the complex plane. In that case (small) deviations from the nominal periodic motion (fixed point) will decrease step after step. The eigenvalues λ are called the Floquet Multipliers and were first used to study the stability of walking by Hurmuzlu (Hurmuzlu and Moskowitz, 1986).

4.2 Example

We will give an example to show what Limit Cycle Walking is and to show that cyclic stability is possible without the constraint of sustained local stability.

Model

The model we will use as an example is the simplest walking model by Garcia et al. (Garcia et al., 1998), shown in Fig. 1. The 2D model consists of two rigid links with unit length *l*, connected at the hip. There are three point masses in the model, one in the hip with unit mass *M* and two infinitesimally small masses *m* in the feet. The model walks down a slope of 0.004 rad in a gravity field with unit magnitude *g*.

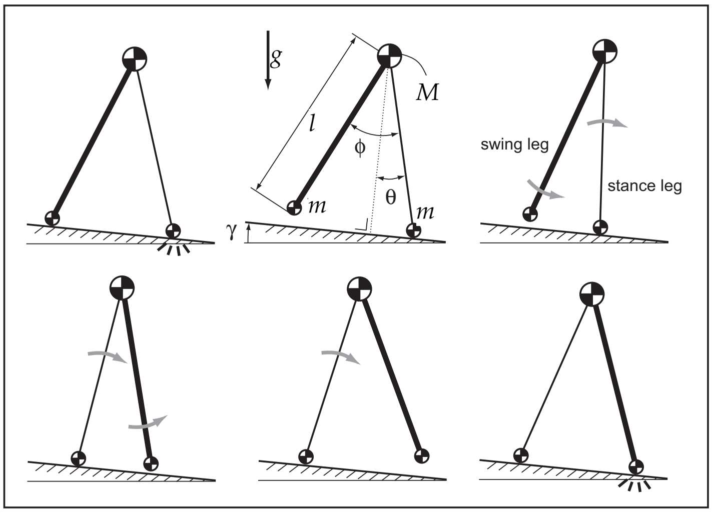


Figure 1. A typical walking step of the simplest walking model. Just after footstrike the swing leg (heavy line) swings forward past the stance leg (thin line) until the swing leg hits the ground and a new step begins. θ is the angle between the stance leg and the slope normal, ϕ is the angle between the two legs, *l* is the leg length, *M* is the hip mass, m is the foot mass, *g* is the gravitational acceleration and γ is the slope angle. Adapted from Garcia et al. (Garcia et al., 1998)

The dynamics of the model consists of two parts. The first part is the continuous dynamics that describes the motion of the stance and swing leg in between footstrikes:

$$\begin{aligned} \ddot{\theta} &= \sin(\theta - \gamma) \\ \ddot{\phi} &= \sin(\phi)\left(\dot{\theta}^2 - \cos(\theta - \gamma)\right) + \sin(\theta - \gamma) \end{aligned} \tag{4}$$

The second part of the dynamics is the discrete impact that describes the footstrike, as this is modeled as a fully inelastic instantaneous collision:

$$
\begin{aligned}
\theta^{+} &= -\theta^{-} \\
\phi^{+} &= -2\theta^{-} \\
\dot{\theta}^{+} &= \cos\left(2\theta^{-}\right)\dot{\theta}^{-} \\
\dot{\phi}^{+} &= \cos\left(2\theta^{-}\right)\left(1-\cos\left(2\theta^{-}\right)\right)\dot{\theta}^{-}
\end{aligned}
\tag{5}
$$

Note that these equations also incorporate the re-labeling of stance and swing leg angles θ and ϕ.
The nominal cyclic motion that results for these dynamic equations is shown in Fig. 2.

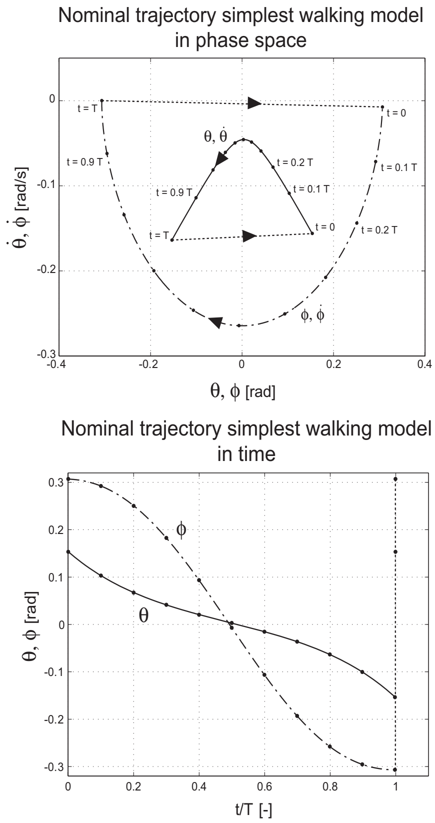


Figure 2. The nominal cyclic motion trajectory of the simplest walking model in phase space (left) and in time (right). In the left plot the solid and dash-dotted lines give the stance leg (θ) and swing leg (ϕ) trajectories respectively, resulting from the continuous dynamics of the model. The dotted lines show the jump conditions that correspond to the discrete footstrike. The large dots along the trajectory indicate the amount of time *t* that has elapsed with increments of 1/10 of the nominal step period *T*. The right plot shows the nominal values of θ (solid) and ϕ (dash-dotted) in time

Cyclic stability

To prove the simplest walking model is cyclically stable, we will perform the stability analysis as described in Section 4.1.

The Poincaré section we choose to use for this cyclic stability analysis is defined as the moment just after heelstrike ($2\theta = \phi$). The `stride function' **S** is the mapping from the system states on the Poincaré section of step *n*: $\mathbf{v_n} = [\theta_n; \dot{\theta}_n; \dot{\phi}_n]$, to the states on the Poincaré section of step *n + 1*: $\mathbf{v_{n+1}} = [\theta_{n+1}; \dot{\theta}_{n+1}; \dot{\phi}_{n+1}]$. First we find the fixed point v^* of the function **S** through a Newton-Raphson search:

$$\mathbf{v}^* = \begin{bmatrix} \theta^* \\ \dot{\theta}^* \\ \dot{\phi}^* \end{bmatrix} = \begin{bmatrix} 0.1534 \\ -0.1561 \\ -0.0073 \end{bmatrix} \tag{6}$$

The monodromy matrix **A** is found by simulating one step for a small perturbation on each of the three states of the fixed point v^*. The eigenvalues λ of matrix **A** for the simplest walking model turn out to be:

$$\lambda = \begin{bmatrix} 0.23 + 0.59i \\ 0.23 - 0.59i \\ 0 \end{bmatrix} \tag{7}$$

These eigenvalues are all within the unit circle in the complex plane, rendering the motion of the simplest walking model cyclically stable.

No sustained local stability

Not only is the motion of the simplest walking model cyclically stable, but we can also prove it is not sustained locally stable around its nominal trajectory. This second proof is needed before we can categorize the simplest walking model as a Limit Cycle Walker.

For this proof, we study the behavior of the motion in the neighbourhood of the nominal trajectory. This behavior is described by the dynamics of the deviations from the nominal trajectory $e_\theta = \theta - \theta_{nom}$ and $e_\phi = \phi - \phi_{nom}$:

$$\begin{aligned} \ddot{e}_\theta &= \sin(\theta_{nom} + e_\theta - \gamma) - \sin(\theta_{nom} - \gamma) \\ \ddot{e}_\phi &= \sin(\phi_{nom} + e_\phi)\left((\dot{\theta}_{nom} + \dot{e}_\theta)^2 - \cos(\theta_{nom} + e_\theta - \gamma)\right) \\ &\quad - \sin(\phi_{nom})\left(\dot{\theta}_{nom}^{\;2} - \cos(\theta_{nom} - \gamma)\right) \\ &\quad + \sin(\theta_{nom} + e_\theta - \gamma) - \sin(\theta_{nom} - \gamma) \end{aligned} \tag{8}$$

We linearize these equations by assuming small deviations Δe_θ and Δe_ϕ. The local stability of this linearized continuous system is evaluated by calculating its poles (roots) along the nominal trajectory depicted in Fig. 2. The resulting root-locus diagram and time plot of the real part of the poles are shown in Fig. 3. There is a pole located in the right-half plane (positive real part) along the whole trajectory, indicating that the simplest walking has no sustained local stability; even more it is not locally stable at any point in time. Thus the motion of the simplest walking model is a clear example of Limit Cycle Walking.

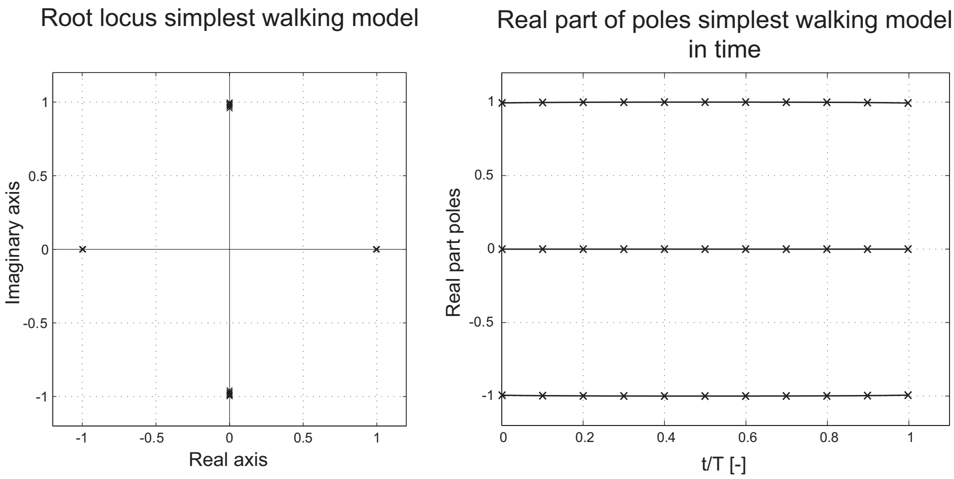


Figure 3. The root locus diagram (left) and time plot of the real part of the poles (right) of the simplest walking model along its nominal trajectory. The pole locations only change slightly throughout the trajectory. The diagrams show that throughout the continuous motion of the simplest walking model there is always a pole in the right-half plane (positive real part); the model is not sustained locally stable

The unnecessary cost of sustained local stability

With this example, we can also show the unnecessary extra constraints that are induced by keeping sustained local stability.

The unstable pole of the simplest walking model corresponds to the inherently unstable motion of the stance leg, being an inverted pendulum. To locally stabilize this motion, the simplest walking model would first need to be able to create a torque τ relative to the ground through a firmly placed foot. Locally stable trajectory tracking (all poles outside the right-hand plane) can be obtained with tracking error feedback with a purely proportional gain K_p only if this feedback gain K_p (normalized for this model) is greater or equal to one:

$$\tau = K_p \cdot e_\theta \quad \text{where } K_p \geq 1 \tag{9}$$

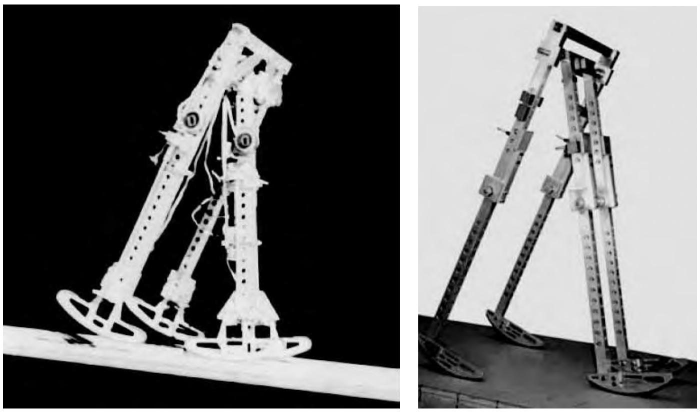
Clearly this shows that the aim for sustained local stability results in unnecessary constraints and unnecessarily high system and control demands. While stable Limit Cycle Walking can be obtained without any effort, sustained local stability requires an extra actuator and tight feedback.

5. State of the Art

There is already a large group of researchers active on Limit Cycle Walking, as we will show in this section. So, one could ask why the concept has not been properly defined earlier. The reason is that many of their robots are derivatives of the so-called Passive Dynamic Walkers (McGeer, 1990a), a subgroup of Limit Cycle Walkers. Passive Dynamic Walking robots will be treated first in this section, followed by actuated Limit Cycle Walkers. Many of these have been referred to as `Passive-based' walking robots, but here we will show that Limit Cycle Walking is a much more accurate label.

5.1 Passive Dynamic Walkers

Passive Dynamic Walking robots are robots that show a perfectly stable gait when walking down a gentle slope without any control or actuation. The simplest walking model that we used in Section 4.2 is a passive dynamic walking model. The stance leg basically is an unstable inverted pendulum and each step is a fall forward. The absence of control clearly places Passive Dynamic Walking as a subgroup of Limit Cycle Walking.
The concept of Passive dynamic walking was introduced in the early 90's by McGeer (McGeer, 1990a), who simulated and built 2D walkers with (McGeer, 1990a) and without knees (McGeer, 1990b) (Fig. 4). As follow-up research, Ruina's group at Cornell University built a 3D “uncontrolled toy that can walk but cannot stand still” (Coleman and Ruina, 1998) (Fig. 5a), the ultimate demonstration of stable Limit Cycle Walking without the existence of any local stability. This group also built a more anthropomorphic 3D prototype with knees and arms (Collins et al., 2001) (Fig. 5b), to show the viability of the concept for the development of humanoid robots. Also in other labs throughout the world passive dynamic walkers have been developed (Mayer et al., 2004; Tedrake et al., 2004a; Ikemata et al., 2006).

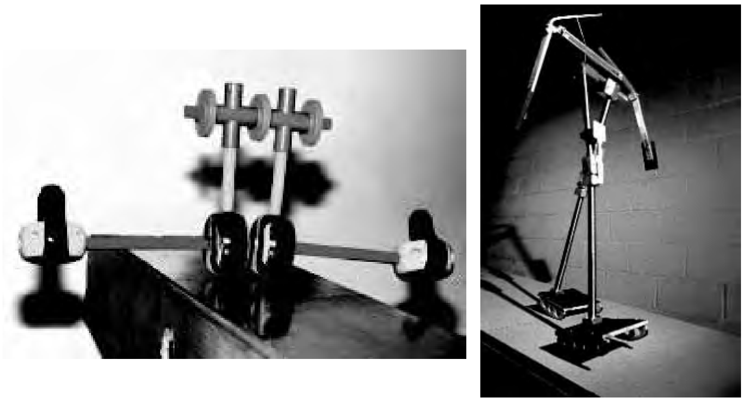


(a) (b)

Figure 4. McGeer's straight legged passive dynamic walker (a) and Cornell University's copy of McGeer's passive dynamic walker with knees (b). Both walkers have 2D dynamics thanks to the double leg pairs (the outer legs form a unit and the inner legs form a second unit)

5.2 Actuated point/arced feet walkers

Perhaps the largest group of typical Limit Cycle Walkers are the actuated robots with point feet or arced feet. They have actuation and control in some of the joints, but the shape of their feet and the resulting point contact or line contact makes these systems underactuated. Although underactuated systems theoretically can be locally stabilized (using the actuated joints, as in the Acrobot (Spong, 1994)), this is not applied in the 2D and 3D robots described here. They depend solely on cyclic stability and thus fall under the category of Limit Cycle Walking.



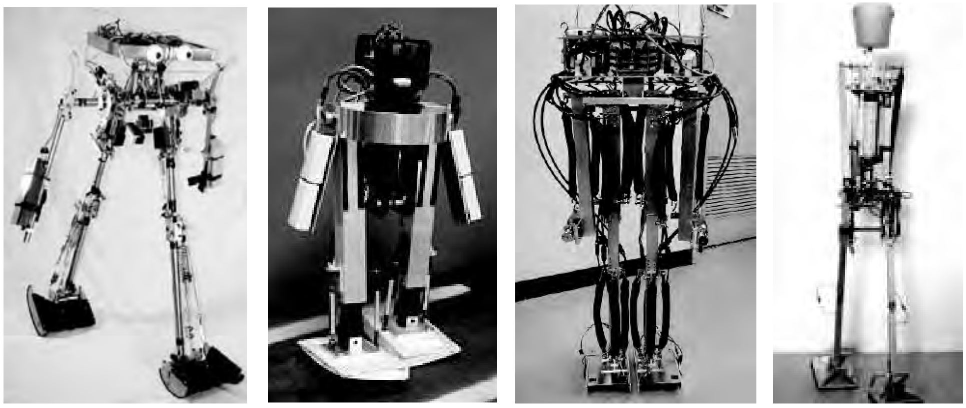
(a) (b)

Fig. 5: Cornell University's "uncontrolled toy that can walk but cannot stand still" (a) and 3D walker with knees and counter-swinging arms (b)

(a) (b) (c) (d) (e)

Figure 6. Two-dimensional actuated point/arced feet walkers: from Carnegie Mellon (a), `Rabbit' from Centre National de Recherche Scientifique (CNRS) and the University of Michigan (b), `Dribbel' from the University of Twente (c), `Cornell Ranger' from Cornell University (d) and `Mike' from Delft University of Technology (e)

The 2D prototypes (Fig. 6) use a four-legged symmetric construction (similar to McGeer's machines) or a guiding boom. Fig. 6a shows a robot with direct drive electric actuators in the hip and knee joints. These weak and highly backdrivable motors make accurate trajectory control practically impossible. Nevertheless, successful walking was obtained with ad-hoc controllers (Anderson et al., 2005), through reinforcement learning (Morimoto et al., 2004) and with CPG control (Endo et al., 2004), demonstrating the potential of the concept of Limit Cycle Walking. At the other end of the spectrum we find the robot `Rabbit' (C.Chevallereau et al., 2003) (Fig. 6b) which has heavily geared electric motors at the hips and knees. The control is based on the concepts of `Hybrid zero dynamics' and `virtual constraints' (Westervelt et al., 2003; de Wit, 2004). It applies tight control on all internal joints, based on the angle of the lower leg with the floor which is left unactuated. As a result, this is a machine with one passive degree of freedom, and so the cyclic stability is calculated with one Floquet multiplier. Between these two extremes, there are currently quite a few 2D Limit Cycle Walking robots (Geng et al., 2006; Dertien, 2006; Wisse and v. Frankenhuyzen, 2003; Wisse et al., 2006), see Fig. 6, including our own pneumatically actuated machines 'Mike' (Wisse and v. Frankenhuyzen, 2003) (Fig. 6e) and 'Max' (Wisse et al., 2006). The great variety of successful controllers and hardware configurations is yet another hint towards the potential of Limit Cycle Walking.



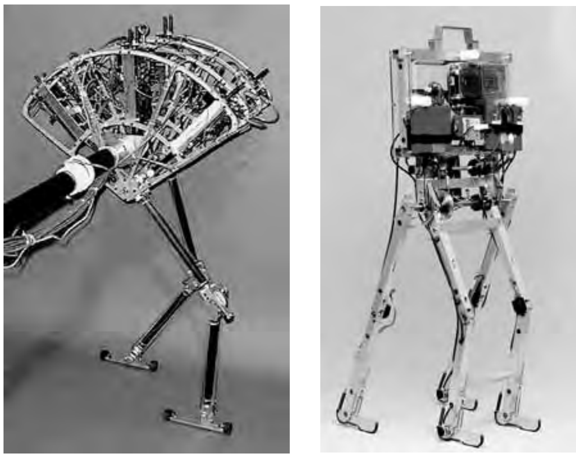
(a) (b) (c) (d)

Figure 7. Three-dimensional actuated arced feet walkers: the `Cornell biped' from Cornell University (a), `Toddler' from Massachusetts Institute of Technology (MIT) (b), `Pneu-man' from Osaka University (c) and `Denise' from Delft University of Technology (d)

There are a few fully three-dimensional (unconstrained) actuated arced feet walkers, as shown in Fig. 7. Cornell University built their `Cornell biped' (Fig. 7a), having an upper body, swinging arms, upper legs, lower legs and arced feet (Collins and Ruina, 2005; Collins et al., 2005). Because of internal mechanical couplings the number of degrees of freedom is only five. The (one DoF) hip joint and knee joints of the `Cornell biped' are fully passive. The only actuation in this machine is a short ankle push-off in the trailing leg just shortly after foot strike at the leading leg. The push-off force is delivered by a spring that is being loaded throughout one step by a small electric motor. `Toddler' (Fig. 7b) is a walker that has been built at MIT (Tedrake et al., 2004a; Collins et al., 2005). It consists of two legs and two arced feet. The (one DoF) hip joint is fully passive, the ankle joint has two degrees of freedom (roll and pitch) which are both activated by position controlled servo motors. Successful gait was obtained with fully feedforward ankle angle trajectories, hand tuned feedback and by applying reinforcement learning (Tedrake et al., 2004b). The `Pneu-man' (Fig. 7c) was developed at Osaka University (Hosoda et al., 2005). It has an upper body, arms, upper legs, lower legs and arced feet. The machine has ten degrees of freedom, two for the arms, two in the hip, two knees and two ankles with two degrees of freedom each. All these joints are actuated by pneumatic McKibben muscles. The `Pneu-man' is controlled through a state machine which is initiated every step at foot strike and subsequently opens and closes pneumatic valves for fixed amounts of time. `Denise' (Fig. 7d) has been developed in our own lab at Delft University of Technology (Wisse, 2005; Collins et al., 2005). It has an upper body, arms, upper legs, lower legs and arced feet. Because of mechanical couplings it only has five internal degrees of freedom. The hip joint is actuated by McKibben muscles while the knee and ankle joints are fully passive.

5.3 Actuated flat feet walkers

We only know of two Limit Cycle Walkers with flat feet, see Fig. 8. `Spring Flamingo' (Fig. 8a) has been built at Massachusetts Institute of Technology and consists of seven links (upper body, upper legs, lower legs and flat feet) (Pratt et al., 2001). All six internal joints are actuated by series elastic actuators, in which springs are intentionally placed in series with geared electric motors (Pratt and Williamson, 1995). This setup allows torque control on all joints. The desired torques for all joints are determined by `Virtual Model Control', an intuitive approach to bipedal walking control, implementing virtual elements such as springs and dampers (Pratt et al., 2001).



(a) (b)

Figure 8. Actuated flat feet Limit Cycle Walkers: `Spring Flamingo' from Massachusetts Institute of Technology (MIT) (a) and `Meta' from Delft University of Technology (b)

`Meta' (Fig. 8b) is a prototype which is currently being used for research in our lab at Delft University of Technology. It has the same links as `Spring Flamingo', but only applies series elastic actuation at the hip joint and the ankles, the knee joints are passive. Various control strategies are currently being applied to this prototype.

`Spring Flamingo' is constrained to 2D by a boom construction and `Meta' by a symmetric construction of its legs. Currently there are no fully 3D flat feet Limit Cycle Walkers in existence. We have adopted this as a research challenge for our lab for the near future.

6. Energy efficiency

With the introduction of the paradigm Limit Cycle Walking we argue that the reduction of unnecessary constraints can result in increased performance. In this section we discuss the performance of Limit Cycle Walking in terms of energy efficiency. We explain how Limit

Cycle Walking is crucial for increasing energy efficiency and we show that State of the Art Limit Cycle Walkers are indeed highly efficient. First we need to explain how the energy efficiency of bipedal walkers is measured to be able to make a fair comparison.

6.1 Measuring energy efficiency

The energy efficiency of bipedal gait is quantified by the specific cost of transport or specific resistance. This dimensionless number c_t gives the amount of energy that the biped uses per distance traveled per weight of the walker:

$$c_t = \frac{\text{Used energy}}{\text{Weight} \cdot \text{Distance traveled}} \tag{10}$$

To distinguish between the efficiency of the total system design and the efficiency of the mechanical design and the applied forces by the controller, the specific energetic cost of transport c_{et} and the specific mechanical cost of transport c_{mt} are defined. The specific energetic cost of transport c_{et} takes into account the total amount of energy used by the system where the specific mechanical cost of transport c_{mt} only takes into account the amount of mechanical work performed by the actuators.

To give a fair comparison between different walkers, the specific cost of transport should be measured at equal normalized walking speeds. Normalized walking speed is given by the Froude number *Fr*, which is the actual walking speed divided by the square root of gravity times leg length: $Fr = v/\sqrt{gl}$.

6.2 Limit Cycle Walking improves energy efficiency

Limit Cycle Walking is energy efficient because it allows the use of zero feedback gains or lower feedback gains than are required for sustained local stability.

When disturbances make a walker deviate from its nominal desired trajectory, high feedback gains forcefully redirect the motion to the nominal trajectory. This often results in excessive active braking by the actuators (negative work) and thus increasing energy consumption. The use of high feedback gains actively fights the natural dynamics of the system at the cost of extra energy expenditure. In contrast, Limit Cycle Walking allows the natural dynamics of a walking system to help ensure convergence to the desired motion which takes away (at least part of) the active braking by the actuators.

In case of uncertain or changing system parameters, the use of zero or low feedback gains in Limit Cycle Walking allows a walker to adapt its gait to the changing natural dynamics. This also results in less energy consumption then forcefully maintaining the originally intended motion which constantly requires the walker to fight its natural dynamics.

6.3 State of the Art energy efficiency

Limit Cycle Walkers show high energy efficiency. For instance, McGeer's first passive dynamic machine has a specific cost of transport of approximately $c_{et} = c_{mt} = 0.025$, walking at a Froude number of $Fr = 0.2$. In comparison, the specific energetic cost of transport c_{et} of humans is about 0.2 based on oxygen consumption, humans' specific mechanical cost of transport c_{mt} is estimated at about 0.05 (Collins et al., 2005). Honda's state of the art humanoid Asimo is estimated to have $c_{et} \approx 3.2$ and $c_{mt} \approx 1.6$ (Collins et al., 2005).

The actuated Limit Cycle Walkers show mechanical costs of transport that are similar to the passive dynamic walkers: c_{mt} = 0.06 for `Dribble, $c_{mt}$ = 0.055 for the `Cornell biped', c_{mt} = 0.08 for `Denise' and $c_{mt}$ = 0.07 for `Spring Flamingo' all at a speed of about Fr = 0.15. The specific energetic cost of transport c_{et} for these machines vary significantly, depending on the effort that was put into minimizing the overhead energy consumption of the system. It ranges from c_{et} = 0.2 for the `Cornell biped' to $c_{et} \approx 5.3$ for `Denise'.

Clearly, the performance of the State of the Art Limit Cycle Walkers shows that energy efficiency is improved through the application of Limit Cycle Walking.

7. Disturbance rejection

This section discusses the performance of Limit Cycle Walking in terms of disturbance rejection, the ability to deal with unexpected disturbances. We explain how Limit Cycle Walking is necessary for increasing disturbance rejection and we show the present disturbance rejection in the State of the Art Limit Cycle Walkers. First we need to explain how the disturbance rejection of bipedal walkers is measured to be able to make a fair comparison.

7.1 Measuring disturbance rejection

The way disturbance rejection is measured in Limit Cycle Walkers unfortunately is quite diverse. A range of disturbance rejection measures exists, relating to the rate at which disturbances are rejected (the largest Floquet multiplier (McGeer, 1990a; Tedrake et al., 2004b)), the maximum allowable size of disturbances (the Basin of Attraction (Schwab and Wisse, 2001)) or a combination of the two (the Gait Sensitivity Norm (Hobbelen and Wisse, 2006)).

The currently most commonly used measure is the maximal floor height variation a walker can handle without falling. A Limit Cycle Walker is perturbed by a single floor height difference (step-up/down) of a specific size to observe whether it is able to recover. The size of this perturbation is varied to establish for which maximal size the walker can still consistently prevent falling. The size of this floor height difference is normalized to the walker's leg length for comparison to other walkers.

7.2 Limit Cycle Walking necessary for large disturbance rejection

Obtaining disturbance rejection of large disturbances in walking is not possible with the application of high feedback gains and sustained local stability as it results in inadmissibly high actuation torques. These overly high desired torques will result in saturation of actuators and loss of full contact between the walker's feet and the floor, violating the necessary condition of local stabilizability for obtaining sustained local stability. The solution to this problem is Limit Cycle Walking as it works with lower actuation torques (lower feedback gains) and it does not depend on full contact between the feet and the floor. The crucial notion is that it is allowed to reject a disturbance over an extended amount of time, as long as falling is avoided.

The application of Target ZMP Control (adjustment of the nominal trajectory) by Honda's ASIMO (Website Honda's ASIMO) is a recognition of the fact that Limit Cycle Walking is necessary for increasing disturbance rejection. We feel that the fact that trajectory

adjustment control is applied, makes ASIMO’s tight trajectory tracking excessive and unnecessary and thus we would recommend the reduction of this constraint altogether.
For Limit Cycle Walkers there are two ways to increase disturbance rejection: (1) minimizing the divergence of motion throughout a step and (2) increasing the stabilizing effect of step-to-step transitions. An exemplary way to minimize motion divergence is the commonly used application of arced feet instead of point feet in Limit Cycle Walkers. A way to increase the stabilizing effect of step-to-step transitions is foot placement, for instance by applying swing leg retraction (Wisse et al., 2005).

7.3 State of the Art disturbance rejection

The disturbance rejection of passive dynamic walkers is limited. The maximal allowable floor height difference has been established on a passive dynamic walking prototype in our lab. This prototype could handle a step-down of ~1.5% of its leg length (Wisse and v. Frankenhuyzen, 2003).
More information on the maximally allowable size of floor height variations is known from the actuated Limit Cycle Walkers. For two-dimensional Limit Cycle Walkers typical values for a single floor height difference (step-down) that they can handle are: ~1% of its leg length for `Max', ~2% for `Mike' and ~4% for `RunBot'. `Spring Flamingo' was able to handle slope angles of 15°, a disturbance that is comparable to a step-down/up of ~9% of its leg length. For three-dimensional walkers the only data that is available is that `Denise' can handle a single step-down disturbance of ~1% of its leg length.
How this State of the Art disturbance rejection compares to other non-Limit Cycle Walkers is hard to say as there is a limited amount of published data available. From personal communication with other robotic researchers we learned that the amount of unexpected floor variation other robotic walkers can handle is probably in the same order of magnitude. This expectation is supported by the fact that high floor smoothness standards are demanded for bipedal robot demonstrations.
The State of the Art disturbance rejection of bipedal robots certainly is small compared to the human ability to handle unexpected disturbances during walking. We expect that Limit Cycle Walking is an essential paradigm for increasing the disturbance rejection of bipedal robots. Proving this in actual prototypes is a main topic of our lab's future work.

8. Versatility

The last aspect of Limit Cycle Walking performance that we will discuss is gait versatility. The versatility of a Limit Cycle Walker is its ability to purposefully perform different gaits. An important aspect of this versatility is the ability to walk at different speeds. We explain how Limit Cycle Walking creates the opportunity to increase versatility and show the versatility of State of the Art Limit Cycle Walkers. First we need to explain how the versatility of bipedal walkers is measured to be able to make a fair comparison.

8.1 Measuring versatility

The versatility of a bipedal walker is largely described by the range of speeds a walker can obtain in three orthogonal directions. In the sagittal/fore-aft direction the range of walking speeds tells whether a walker is able to walk at high speeds and low speeds, but also if it is able to come to a standstill (zero speed). In the other two directions (lateral/left-right and vertical) it is the ratio between the speed in that direction and the sagittal walking speed that is of importance. In the lateral direction this defines the rate at which a walker can turn (the turn radius it can obtain) or, when the ratio is infinite, whether it can walk sideways. In the vertical direction it defines what types of slopes (or stairs) the walker can handle purposefully. To make walking speeds comparable between different walkers the speed is generally normalized by giving the dimensionless Froude number: $Fr = v/\sqrt{gl}$.

8.2 Limit Cycle Walking increases speed range

Limit Cycle Walking enables walking at high speeds because it uses the more frequent and more violent step-to-step transitions at high speed as a way to create stability instead of having to fight them as disturbances.

In case a bipedal walker applies sustained locally stabilizing control around its nominal trajectory, the step-to-step transitions generally are mainly a source of deviations from the nominal motion which have to be rejected by the trajectory controller. For increasing walking speed, these induced deviations tend to grow in size and occur at a higher frequency, which increases the demands on the trajectory controller. Eventually, the obtainable walking speed is limited by the control bandwidth.

In Limit Cycle Walking this control bandwidth limitation on walking speed does not exist as it does not depend on continuous stabilizing control to reject deviations. Step-to-step transitions are the main source of stabilization and the `bandwidth' of this stabilizing effect automatically increases with increasing walking speed.

8.3 State of the Art versatility

Some of the State of the Art Limit Cycle Walkers have shown to be able to walk at different walking speeds: 'Rabbit' shows a speed range from about Fr = 0.15 to Fr = 0.3, `Toddler' from 0 (it is able to start from standstill) to about $Fr$ = 0.09 and the relatively fast and small `RunBot' from Fr = 0.25 to Fr = 0.5. In comparison Honda's Asimo has a speed range from 0 to Fr = 0.3 and humans from 0 to about Fr = 1.

Research with Limit Cycle Walkers focusing on the ability to purposefully perform turns or walk stairs has gotten little attention so far. Only the prototype `Cornell Ranger' is able to make turns with a minimal radius of about 25 meters. None of the State of the Art Limit Cycle Walkers has shown the ability to purposefully walk up or down stairs.

The State of the Art versatility of Limit Cycle Walkers is quite limited at present in spite of the fact that the paradigm is promising on this aspect. Showing that Limit Cycle Walking prototypes can have a large versatility is another main topic of our lab's future work.

9. Conclusion

In this chapter we have introduced the paradigm `Limit Cycle Walking'. This paradigm has been used for some time by a group of bipedal robotics researchers, but the concept had not been properly defined before:

Limit Cycle Walking is a nominally periodic sequence of steps that is stable as a whole but not locally stable at every instant in time.

Limit Cycle Walking releases the unnecessary stability constraint of sustained local stability that is often applied in bipedal robots. It only requires the much less restrictive cyclic stability. This gives extra freedom for finding better walking performance, it is expected that:

- Limit Cycle Walking increases energy efficiency because it does not apply high feedback gains that fight the natural dynamics of the system in case of disturbances or uncertain parameters.
- Limit Cycle Walking is necessary to increase disturbance rejection because large disturbances tend to violate the constraints that are necessary to enforce local stability.
- Limit Cycle Walking increases versatility by enabling higher walking speeds; the more frequently and violently occurring step-to-step transitions are inherently used for stabilization in contrast to being an increasing source of unwanted disturbances demanding higher control bandwidth.

The increase of energy efficiency has been demonstrated by the State of the Art Limit Cycle Walkers. The expected increase in disturbance rejection and versatility has not yet been shown in existing bipedal walkers. Doing this is the main goal of our lab's future work.

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